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Application of Dynamic Economic-Mathematical Modeling in Optimization Problems in Banking

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Abstract. The topical issues of optimizing the management of banking in order to increase the economic efficiency and competitiveness of the bank are discussed. To solve the problem of optimizing the management of the number of employees and sales of the Bank's Retail Unit, it is proposed to use dynamic economic and mathematical modeling that takes into account the presence of control actions and a vector quality criterion. The criterion of quality is the indicator Cost Income Ratio (CIR) — the ratio of operating costs to operating income. CIR is actively used around the world to assess the effectiveness of the bank by investors, shareholders, rating agencies. The report presents the main stages in the development of the considered dynamic model. An algorithm for solving the posed optimization problem is proposed. Based on the developed computer modeling system and the results of computer modeling, the optimal solution for the optimization problem under consideration was chosen. The paper illustrates all the results obtained and analyzes them. The developed model and method for solving the problem of optimizing the management of the number of employees and sales of the Bank's Retail Unit can serve as a basis for the development, creation and application of appropriate computer information systems to support the adoption of managerial decisions in banking.

INTRODUCTION

The issues of managing the efficiency of banking deserve special attention. In the situation of unstable financial behavior of the external environment, taking into account constantly changing requirements of "regulators", the management of any bank seeks to minimize risks and increase net operating income of its business. Along with this, the industry is developing dynamically, modern technologies and electronic sales channels are being introduced. High competition in the industry remains, despite the reduction in the number of participants. In response to constantly growing customer demands, both in terms of speed and the quality of service, representatives of the banking services market are forced to improve their activities ahead of schedule in order to remain firmly in the banking services market. Micro-credit organizations, as well as other companies that provide financial services, but are not banks at the same time, compete with banks. Also, the tools of competition are the constant refinement of strategy and tactical approaches to management in credit institutions. An important influence is rendered also by marketing approaches to attraction of clients, and also thought over advertising.

The most important direction to achieve high efficiency of banking business management is the management of available resources, including staff. Given the rapid development of remote service channels, the question arises about the need to maintain a network of additional offices of the bank, regardless of whether they are on lease or in ownership. Along with the contents of the network, the issue of the need to keep a significant number of employees working with clients and reducing their numbers requires solution. Existing staff should be used as efficiently as possible, establish sales standards, monitor deviations from planned indicators. Most often, decisions that determine the direction of banking business development are taken on the basis of expert opinion of the members of the bank's

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management and other persons on whom decision-making depends. Less often, decisions are made on the basis of data from individual calculations of the financial result of ongoing activities. The availability of an information and analytical decision support system significantly improves the efficiency, which allows access to qualitative data for analysis as quickly and efficiently as possible. This will allow you to quickly and correctly assess all the consequences that await business as a result of taking a decision and choose the most effective of them.

The system of decision support, both managerial and financial, includes a set of related methods, software tools and hardware that automatically implement a set of various reports containing the necessary information for decision making. The most modern systems can offer variants of such solutions.

The best efficiency allows us to achieve the application of economic and mathematical models and methods, including machine learning technologies, neural networks for the analysis of Big Data [1]–[3]. Using the latest approaches allows you to get results that were not incorporated into the original algorithms, since the system itself learns over time as a result of the implementation of banking activities.

CONSTRUCTION OF THE ECONOMIC-MATHEMATICAL MODEL

Consider the parameters of the process of making managerial decisions in the Bank's Retail Unit activities by the example of entering the number of employees of the Retail Unit and setting sales standards for various categories of employees in terms of the impact of this measure on the dynamics of retail product portfolios and the profit of the Bank's Retail Unit. Similar models were considered in a more simplified form in [4]–[5] .

To form an economic-mathematical model of the decision-making process in banking, we introduce the following notation:

n is the total number of basic bank "portfolio" products (we will refer to "portfolio" products those whose condition significantly affects the value (in selected units) of the criterion (criteria) for the quality of the existing bank portfolio, for example, consumer loans, term deposits, bank cards, etc.; $n \in \mathbb{N}$; where $\mathbb{N}$ is here and below, the set of all natural numbers);

m is the total number of types of positions of employees implementing these products (for each of these positions, certain functions for the sale of certain products are assigned; $m \in \mathbb{N}$);

$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbb{R}^n$ is the vector characterizing the volume of the portfolio of each of the banking products in the period of time t in thousands of rubles ($t \in \overline{0, T-1} = \{0, 1, 2, \ldots, T-1\}$; $T \in \mathbb{N}$), for which each i-th coordinate $x_i(t)$ is the value of the portfolio volume of the i-th type of banking product ($i \in \overline{1, n}$); here and below for $k \in \mathbb{R}^k$, $\mathbb{R}^k$ is the k-dimensional Euclidean vector space of column vectors;

$y(t) = (y_1(t), y_2(t), \ldots, y_n(t))' \in \mathbb{R}^m$ is the vector characterizing the number of different categories of employees in the bank (number of people) in a period of time t ($t \in \overline{0, T-1}$), where each j-th coordinate $y_j(t)$ is the value of the staff number of the j-th type of position ($j \in \overline{1, m}$);

$A(t) = \|a_{ij}(t)\|$, $i \in \overline{1, n}$, $j \in \overline{1, m}$ is the matrix of sales standards per month in the period of time t ($t \in \overline{0, T-1}$), $a_{ij}(t)$ is the normative number of products sold i-th type of employee j-th category ($i \in \overline{1, n}$, $j \in \overline{1, m}$);

$z = (z_1, z_2, \ldots, z_n)' \in \mathbb{R}^n$ is the vector of labor costs for the sale of each type of product, taking into account the sales funnel (i.e. the time spent consulting all customers, regardless of their decision to purchase the product), minutes;

$H = (h_1, h_2, \ldots, h_n)' \in \mathbb{R}^n$ is the vector of the monthly repayment coefficients for each of the product types in the portfolio, %;

$S = (s_1, s_2, \ldots, s_n)' \in \mathbb{R}^n$ is the vector of average values of products sales, thousand rubles;

$u(t) = (u_1(t), u_2(t), \ldots, u_m(t))' \in \mathbb{R}^m$ is the vector of input of the number of each category of employees (number of people), in the period of time t ($t \in \overline{0, T-1}$), in which each j-th coordinate $u_j(t)$ is the value of the number of input staff members of the j-th type of position ($j \in \overline{1, m}$).

Based on the parameters introduced, the dynamics of the process under consideration will be described by the following system of recurrence equations:

$$\begin{cases} y_j(t+1) = y_j(\tau_t) + u_j(\tau_t), \ u_j(0) = 0, \ \tau_t = \tau \cdot E(\frac{t}{\tau}), \\ x_i(t+1) = x_i(t) - h_i \frac{x_i(t)}{100} + s_i \sum\limits_{j=1}^{m} a_{ij}(t)[y_j(t) + u_j(t)], \\ \qquad t \in \overline{0, T-1}, \ i \in \overline{1, n}, \ j \in \overline{1, m}, \end{cases} \tag{1}$$

where the following designations are accepted:

$x(t+1) = (x_1(t+1), x_2(t+1), \ldots, x_n(t+1))' \in \mathbb{R}^n$ is the vector of the volume of portfolios of banking products in the time period $(t+1)$;

$y(t+1) = (y_1(t+1), y_2(t+1), \ldots, y_m(t+1))' \in \mathbb{R}^m$ is the vector of the number of different categories of employees (number of people) in time period $(t+1)$;

$\tau \in \mathbb{N} : \ \tau \leq T$;

$E : \mathbb{R}^1 \to \mathbf{Z}$ is the integer part of the number.

Note that the generated system (1) allows to simulate the dynamics of the multi-step process of managing the portfolio structure of the bank's retail products depending on the given initial conditions and the choice of specific realizations of control actions-changes in the number of employees and establishment of sales standards for them.

The above described vector of input the number of employees of the Bank's Retail Unit $u(t) = (u_1(t), u_2(t), \ldots, u_m(t))' \in \mathbb{R}^m$ and the sales standards matrix $A(t) = \|a_{ij}(t)\|$, $i \in \overline{1,n}$, $j \in \overline{1,m}$ in the time period t $(t \in \overline{0, T-1})$, there are control actions in the system for which the following constraints:

$$\begin{gathered}
u(t) \in U_1(t, y(t-1), y(t)) \subset \mathbb{R}^m, \\
U_1(t) = \{u(t) : \ u(t) \in \{u^{(1)}(t), u^{(2)}(t), \ldots, u^{(N_t)}(t)\}\} \subset \mathbb{R}^m, \ N_t \in \mathbb{N}, \\
y(t) = (y_1(t), y_2(t), \ldots, y_m(t))' \in \mathbb{R}^m, \\
(\forall j \in \overline{1,m}) \bigwedge (\forall k \in \overline{1,N_t}) : \ |u_j^k(t)| \leq 0,1 \cdot y_j(t-1); \ \sum_{j=1}^{m} y_j(t) \leq 10000;
\end{gathered} \tag{2}$$

$$\begin{gathered}
A(t) \in A_1(t) \subset \mathbb{R}^{n \times m}, \\
A_1(t) = \Big\{A(t) : \ A(t) \in \{A^{(1)}(t), A^{(2)}(t), \ldots, A^{(M_t)}(t)\} \subset \mathbb{R}^{n\times m}, \ M_t \in \mathbb{N} \ , \\
\forall k \in \overline{1, M_t} : \ A^{(k)}(t) = \|a_{ij}^{(k)}(t)\|, \ i \in \overline{1,n}, \ j \in \overline{1,m}; \\
((\forall i \in \overline{1,n}) \bigwedge (\forall \ j \in \overline{1,m}) : \ a_{ij}^{(k)}(t) \geq 0) \bigwedge (T_{min} \leq \sum_{i=1}^{n} [a_{ij}^{(k)}(t) \cdot z_i] \leq T_{max}); \\
((\forall i \in \overline{1,n}) \bigwedge (\forall j \in \overline{1,m}) : \ a_{ij}^{(k)}(t) \geq 0) \bigwedge (\forall a_{ij}^{(k)}(t) > 0) : \\
a_{ij}^{(k)}(t) \cdot z_i \geq 0,1 \cdot \sum_{i=1}^{n} [a_{ij}^{(k)}(t) \cdot z_i], z_i \in \mathbb{R}^1 \},
\end{gathered} \tag{3}$$

where T_{min} is the value of the minimum allowable time for sale of banking products per month, minutes, T_{max} is the value of the standard working time per month, minutes; N_1 is the number of permissible controls $u(t)$ in the time period t $(N_1 \in \mathbb{N})$; M_1 is the number of admissible $A(t)$ matrices of sales standards in the period of time t $(M_1 \in \mathbb{N})$.

In addition, for all $t \in \overline{0, T-1}$ the following predetermined phase constraints must also be satisfied:

$$\begin{gathered}
x(t) \in X_1(t), \ y(t) \in Y_1(t), \\
X_1(t) = \{x(t) : \ x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbb{R}^n, \ \forall i \in \overline{1,n} : \ x_i(t)) \geq 0\}, \\
Y_1(t) = \{y(t) : \ y(t) = (y_1(t), y_2(t), \ldots, y_m(t))' \in \mathbb{R}^m, \ \forall i \in \overline{1,m} : \ y_i(t) \geq 0\}.
\end{gathered} \tag{4}$$

For the formed discrete dynamic system (1)–(4) below, we consider a number of indicator (functional) — quality criteria (objective functions) that estimate the results of possible realizations of the process of managing the number of employees of the Retail Bank Unit and their sales system.

We fix the pair $\overline{u}(\cdot) = \{u(\cdot), A(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$, that forms an admissible program control on the time interval $\overline{0,T}$, where: $u(\cdot) = \{u(t)\}_{t \in \overline{0,T-1}}$; $A(\cdot) = \{A(t)\}_{t\in\overline{0,T-1}}$; $\forall t \in \overline{0, T-1} : \ \overline{\mathbf{U}}(t) = U_1(t, y(t-1), y(t)) \times A_1(t)$, $\overline{\mathbf{U}}(0) = U_1(0) \times A_1(0)$; $\overline{\mathbf{U}}(\cdot) = \{\overline{\mathbf{U}}(t)\}_{t\in\overline{0,T-1}}$ is the set of all admissible program control actions on the interval time $\overline{0,T}$ in the dynamical system (1)–(4).

Let $\overline{x}(T) = \varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}(\cdot))$ be the final state (state at time T) of the trajectory $x(\cdot) = \{x(t)\}_{t\in\overline{0,T}}$ of the phase vector $\overline{x}(t) = \{x(t), y(t)\}' \in \mathbb{R}^{m+n}$ $(t \in \overline{0,T})$ of the system (1)–(4), describing the dynamics of the process of optimizing the complex program management of the number of employees of the Bank's Retail Unit and the sales system for the time interval $\overline{0,T}$, corresponding to the set $\{\overline{x}(0), \overline{u}(\cdot)\} \in \overline{X}(0) \times \overline{\mathbf{U}}(\cdot)$, where $\overline{X}(0) = \{x(0)\} \times \{y(0)\}$.

Then for all admissible realizations of the sets $\{\overline{x}(0), \overline{u}(\cdot)\} \in \overline{X}(0) \times \overline{\mathbf{U}}(\cdot)$ the quality of the control process in the system (1)–(4) describing the dynamics of the process of optimizing the complex software management of the number of employees of the Bank's Retail Unit and their sales system in the time interval $\overline{0,T}$, can be estimated by the following terminal functional (process quality indicator).

Functional Cost Income Ratio (CIR) — the ratio of operating costs to operating income.

We introduce additional parameters into the generated model:

$v = (v_1, v_2, \dots, v_m)' \in \mathbb{R}^m$ is the vector of the wage value for each category of employees, thousand rubles;

$r = (r_1, r_2, \dots, r_n)' \in \mathbb{R}^n$ is the vector of interest and transfer income rates for each type of portfolio on an annual basis, %;

$c = (c_1, c_2, \dots, c_n)' \in \mathbb{R}^n$ is the vector of interest and transfer costs for each type of portfolio on an annualized basis, %;

q is the value of monthly administrative and business expenses excluding labor costs, thousand rubles.

Then the objective function $\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot))$ of optimization of complex program management by the number of employees of the Bank's Retail Unit and the sales system reflecting the value of the indicator Cost Income Ratio (CIR) in the time interval $\overline{0,T}$ will be calculated as follows:

$$\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) = CIR(T) = \frac{T \cdot q + \sum_{t=1}^{T} \sum_{j=1}^{m} v_j \cdot y_j(t)}{\frac{1}{24 \cdot 100} \sum_{t=1}^{T} \sum_{i=1}^{n} (x_i(t) + x_i(t-1)) \cdot (r_i - c_i)} =$$

$$= F_{\overline{0,T}}(\varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}(\cdot))) = F_{\overline{0,T}}(\overline{x}(T)), \tag{5}$$

where $CIR(t)$ is the ratio of operating costs to the operating income of the Bank's Retail Unit, %, for a period of time t ($t \in \overline{1,T}$); $CIR(0) = 0$.

Then the task of optimizing the complex program management of the number of employees of the Bank's Retail Unit and their sales system can be formulated as follows.

For the process of optimizing the program management of the number of employees of the Bank's Retail Unit and the sales system described by the discrete dynamic economic-mathematical model (1)–(4) and the given initial phase vector $\overline{x}(0) = \{x(0), y(0)\}' \in \mathbb{R}^{m+n}$, considered for a given time interval $\overline{0,T}$, it is required to find a pair of admissible program controls $\overline{u}^{(e)}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$ on this interval of time so that the value of the terminal objective function $\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot))$, defined by the relation (5), was a minimal in comparison with all other admissible values for it, corresponding to other pairs of admissible program controls $\overline{u}(\cdot) = \{u(\cdot), A(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$, that is, the following optimality condition holds:

$$\mathbf{\Phi}^{(e)}(T) = \mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}^{(e)}(\cdot)) = \max_{\overline{u}(\cdot)=\{u(\cdot),A(\cdot)\}\in\overline{\mathbf{U}}(\cdot)} \mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) =$$

$$= \max_{\overline{u}(\cdot)=\{u(\cdot),A(\cdot)\}\in\overline{\mathbf{U}}(\cdot)} F_{\overline{0,T}}(\varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}(\cdot))) = F_{\overline{0,T}}(\varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}^{(e)}(\cdot))) =$$

$$= F_{\overline{0,T}}(\overline{x}^{(e)}(T)) = F_{\overline{0,T}}(x^{(e)}(T), y^{(e)}(T)) = F(\overline{x}^{(e)}(T)) = F(x^{(e)}(T), y^{(e)}(T)) = \mathbf{F}^{(e)}(T), \tag{6}$$

where $\overline{x}^{(e)}(T) = \varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}^{(e)}(\cdot)) = \{x^{(e)}(T), y^{(e)}(T)\}$.

The output results of optimization of the process of program management of the number of employees of the Bank's Retail Unit is a set of data $\{u^{(e)}(\cdot), A^{(e)}(\cdot), \mathbf{F}^{(e)}(T)\}$, where $u^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t\in\overline{0,T-1}}$ is the array of optimal values of the input quantity vector for the staff of the bank's retail unit, $A^{(e)}(\cdot) = \{A^{(e)}(t)\}_{t\in\overline{0,T-1}}$ is the array of optimal values for the matrices of the standards for sales of banking products by employees of the Bank's Retail Unit, $\mathbf{F}^{(e)}(T)$ is the optimal value of the quality functional at the final time T.

ALGORITHM FOR SOLVING THE PROBLEM OF OPTIMAL PROGRAM CONTROL

Consider the algorithm for solving the formulated problem of optimal program management by the number of employees of the Bank's Retail Unit and their sales system on a concrete practical example, which is a special case of the general economic-mathematical model (1)–(6).

Let $n = 8$, $m = 7$ and the parameters of the system (1)–(6) be described on the time interval $\overline{0,T}$ in the time period t ($t \in \overline{0,T}$) by the following data:

$\tau = 6$, i.e. the number of employees of the bank can vary with a periodicity of 6 months — no more than once every six months;

$x(t) = (x_1(t), x_2(t), \ldots, x_8(t))' \in \mathbb{R}^8$;

$x_1(t)$ is the volume of the portfolio of debt on consumer loans;

$x_2(t)$ is the volume of the portfolio of debt on housing loans;

$x_3(t)$ is the volume of the auto loan portfolio;

$x_4(t)$ is the volume of the credit card debt portfolio;

$x_5(t)$ is the volume of the term deposit portfolio — deposits of individuals;

$x_6(t)$ is the volume of the portfolio of balances on the accounts of personal debit bank cards;

$x_7(t)$ is the volume of the portfolio of balances on securities of individuals;

$x_8(t)$ is the volume of the portfolio of balances on the accounts of salary bank cards;

Number of employees:

$y(t) = (y_1(t), y_2(t), \ldots, y_7(t))' \in \mathbb{R}^7$;

$y_1(t)$ is the number of sales managers;

$y_2(t)$ is the number of managers of mortgage lending;

$y_3(t)$ is the number of specialists in direct sales;

$y_4(t)$ is the number of senior specialists in direct sales;

$y_5(t)$ is the number of managers for salary projects;

$y_6(t)$ is the number of managers working with partners;

$y_7(t)$ is the number of specialists in the service of private individuals.

We know the initial state at time $t = 0$ of the phase vector $\overline{x}(0) = \{x(0), y(0)\}' \in \mathbb{R}^{15}$.

It is assumed that the restriction of (2) to the discrete control value $u(t) = (u_1(t), u_2(t), \ldots, u_7(t))' \in \mathbb{R}^7$ for each $t \in \overline{0, T-1}$ has the following concrete form:

$$u(t) \in \mathbf{U}_1(t, y(t-1), y(t)) = \Big\{u(t): \; u(t) \in \{u^{(1)}(t), u^{(2)}(t), u^{(3)}(t)\} \subset \mathbb{R}^7, \; \forall k \in \overline{1,3},$$

$$u^{(k)}(t) = (u_1^{(k)}(t), u_2^{(k)}(t), \ldots, u_7^{(k)}(t))' \in \mathbb{R}^7, y(t) = (y_1(t), y_2(t), \ldots, y_7(t))' \in \mathbb{R}^7,$$

$$y(t-1) = (y_1(t-1), y_2(t-1), \ldots, y_7(t-1))' \in \mathbb{R}^7,$$

$$\forall j \in \overline{1,7}, \; u_j^{(k)}(t) \in \{-u_j^{*(k)}(t); 0; u_j^{*(k)}(t)\},$$

$$u_j^{*(k)}(t) = 0,1 \cdot y_j(t-1), \; \sum_{j=1}^{7} y_j(t) \leq 10000, \; y_j(t) = y_j(t-1) + u_j(t-1),$$

$$t \geq 2: \; y_j(t-1) = y_j(t-2) + u_j(t-2)\,\},$$

i.e. $N_t = 3$ for all $t \in \overline{0, T-1}$. In addition, the control $u(t)$ determines the first component of the generalized control action $\overline{u}t) = \{u(t), A(t)\} \in \overline{\mathbf{U}}(t)$, where $u(t) = u^{(k)}(t) \in \mathbf{U}_1(t)$ for all $t \in \overline{0, T-1}$ and $k \in \overline{1,3}$, and it is assumed that for each $k \in \overline{1,3}$ a single initial control value $u(0) = u^{(k)}(0) = \mathbf{0}_7 \in \mathbf{U}_1(0)$ is given, i.e. $\mathbf{U}_1(0) = \{u(0)\}$ is a one-element set, where $\mathbf{0}_7 = (0, 0, \ldots, 0)'$ is the zero vector of the space $\mathbb{R}^7$.

In accordance with the restriction (3), we also define a sequence of matrices $\underline{A_1(t) = \{A(t) : A(t) \in \{A^{(1)}(t), A^{(2)}(t), A^{(3)}(t)\}\}}$, consisting of only three matrices, i.e. with $M_t = 3$, for all $t \in \overline{0, T-1}$, and such that for each time period t ($t \in \overline{0, T-1}$), the corresponding matrix $A^{(k)}(t) \in A_1(t) (k \in \overline{1,3})$ defines the second component of the generalized control action $\overline{u}(t) = \{u(t), A(t)\} \in \overline{\mathbf{U}}(t)$, where $A(t) = A^{(k)}(t)$. This means that you can simulate the process of program control under different matrices. It is assumed that for each $k \in \overline{1,3}$ a single initial value of the sales norm matrix $A(0) = A^{(k)}(0) \in A_1(0)$ is given. Then, the algorithm for modeling the solution of the problem of optimal program control for the number of personnel of a Bank's Retail Unit in the presence of an objective function of the form (5) can be represented as the implementation of the following sequence of actions.

Step 0. Forming the initial data.

0.1. A natural number $T \in \mathbb{N}$ is introduced, which determines the period of optimization of the control of the process under consideration.

0.2. For $t = 0$ the initial value of the phase vector $\overline{x}(0) = \{x(0), y(0)\}' \in \mathbb{R}^{15}$ is formed.

0.3. For $t = 0$ we form a vector $u(0) = (u_1(0), u_2(0), \ldots, u_7(0))' \in \mathbb{R}^7$ which, in accordance with the constraint (2), defines a one-element set $\mathbf{U}_1(0) = \{u(0)\}$.

0.4. For $t = 0$ a matrix $A(0)$ is formed, which, in accordance with the constraint (3), defines a singleton set $A_1(0) = \{A(0)\}$.

0.5. A real numerical value $\mathbf{F}^{(e)}(T) = -10^{10}$ is formed.

Step 1. Formation of a set of admissible control actions.

A finite set of all admissible program controls is formed on the time interval $\overline{0,T}$:

$$\overline{\mathbf{U}}(\cdot) = \Big\{\overline{u}(\cdot) : \ \overline{u}(\cdot) = \{u(\cdot), A(\cdot)\}, \ u(\cdot) = \{u(t)\}_{t\in\overline{0,T-1}}, \ A(\cdot) = \{A(t)\}_{t\in\overline{0,T-1}}, \ \forall t \in \overline{0,T-1},$$

$$u(t) = (u_1(t), u_2(t), \ldots, u_7(t))' \in \mathbb{R}^7, \ A(t) = \|a_{ij}\|_{\substack{i\in\overline{1,7}\\ j\in\overline{1,8}}} \in \mathbb{R}^{7\times 8},$$

$$y(t) = (y_1(t), y_2(t), \ldots, y_7(t))' \in \mathbb{R}^7, \ y(t-1) = (y_1(t-1), y_2(t-1), \ldots, y_7(t-1))' \in \mathbb{R}^7,$$

$$\forall j \in \overline{1,7}, \ u_j(t) \in \{-u_j^*(t); 0; u_j^*(t)\},$$

$$u_j^*(t) = 0,1 \cdot y_j(t-1), \ \sum_{j=1}^{7} y_j(t) \le 10000, \ y_j(t) = y_j(t-1) + u_j(t-1),$$

$$t \ge 2 : \ y_j(t-1) = y_j(t-2) + u_j(t-2), \ A(t) \in \{A^{(1)}(t), A^{(2)}(t), A^{(3)}(t)\}\ \},$$

$$\overline{\mathbf{U}}(0) = U_1(0) \times A_1(0) = \{u(0), A(0)\},$$

which consists of $K_t = 9^{7\times(T-1)}$ elements $\overline{u}^{(k)}(\cdot), \ k \in \overline{1, K_t}$.

Step 2. Formation of the final phase vector of the system.

2.1. The beginning of cycle 1 with integer variable $k \in \overline{1, K_T}$.

2.2. The control action $\overline{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$ is formed, where $\overline{u}^{(k)}(0) = \{u(0), A(0)\}$.

2.3. In accordance with the equations of dynamics (1), the value of the phase vector $\overline{x}^{(k)}(T) = \{x^{(k)}(T), y^{(k)}(T)\} = \varphi_{\overline{0,T}}(T; \overline{x}^{(k)}(0), \overline{u}^{(k)}(\cdot))$, where $\overline{x}^{(k)}(0) = \{x^{(k)}(0), y^{(k)}(0)\}' = \{x(0), y(0)\}' \in \mathbb{R}^{15}$.

Step 3. Forming the value of a terminal functional.

3.1. For the phase vector $\overline{x}^{(k)}(T) = \{x^{(k)}(T), y^{(k)}(T)\} = \varphi_{\overline{0,T}}(T; \overline{x}^{(k)}(0), \overline{u}^{(k)}(\cdot))$, corresponding to the k-th program control $\overline{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$, in accordance with the constraint (5), the value of the functional $F(\overline{x}^{(k)}(T))$ is calculated by the formula:

$$F(\overline{x}^{(k)}(T)) = F(x^{(k)}(T), y^{(k)}(T)).$$

3.2. If $F(\overline{x}^{(k)}(T)) > \mathbf{F}^{(e)}(T)$, then go to point 3.3, otherwise — to 3.5.

3.3. Formed values: the real variable $\mathbf{F}^{(e)}(T) = F(\overline{x}^{(k)}(T))$.

3.4. An array of control actions is formed $\overline{u}^{(e)}(\cdot) = \overline{u}^{(k)}(\cdot) = \{\overline{u}^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$.

3.5. End of cycle 1 with integer variable $k \in \overline{1, K_T}$.

Step 4. Display simulation results.

As a result of the implementation of the previous steps of this algorithm, the following parameters are calculated:

4.1. The real number $\mathbf{F}^{(e)}(T)$ is the optimal value of the quality criterion for the process under consideration.

4.2. The real array $\overline{u}^{(e)}(\cdot) = \{\overline{u}^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$ is the optimal program control for the process under consideration, where $\overline{u}^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t\in\overline{0,T-1}}$; $A^{(e)}(\cdot) = \{A^{(e)}(t)\}_{t\in\overline{0,T-1}}$; $\overline{\mathbf{U}}(\cdot) = \{\overline{\mathbf{U}}(t)\}_{t\in\overline{0,T-1}}$.

4.3. The real array $\overline{x}^{(e)}(\cdot) = \{\overline{x}^{(e)}(\cdot), y^{(e)}(\cdot)\} = \{\{x^{(e)}(t)\}_{t\in\overline{0,T-1}}, \ \{y^{(e)}(t)\}_{t\in\overline{0,T-1}}\}$ is the optimal trajectory for process, i.e. $\overline{x}^{(e)}(\cdot) = \varphi_{\overline{0,T}}(\cdot; \overline{x}^{(e)}(0), \overline{u}^{(e)}(\cdot)), \overline{x}^{(e)}(0) = \{x^{(e)}(0), y^{(e)}(0)\}' \in \mathbb{R}^{15}$.

4.4. The results are displayed in a user-friendly form, for example, in the form of graphs or tables.

End of algorithm

Note that the elements $\overline{u}^{(e)}(\cdot) = \{\overline{u}^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot)$ obtained with the help of the described algorithms, and $\mathbf{F}^{(e)}(T)$ satisfy the optimality condition (6); they are the solution of the considered problem of optimization of complex program management by the number of employees of the Bank's Retail Unit and the system of their sales in the presence of an objective function of the form (5).

FORMATION OF INITIAL DATA FOR COMPUTER MODELING

Let us consider the problem of optimal programmatic control of the number of personnel and sales of the Bank's Retail Unit, described by relations (1)–(6) with the following initial conditions:

The initial state (for $t = 0$) of the phase vector $\overline{x}(0) = \{x(0), y(0)\}' \in \mathbb{R}^{15}$ and the matrix of sales standards $A(0)$ is known. It is assumed that for the control process under consideration a sequence of matrices $\{A(t)\}_{t\in\overline{0,T-1}}$, is given such that for the period t $(t \in \overline{0, T-1})$, the corresponding matrix $A(t)$ defines the second component of the control action. This means that it is possible to model the process of program control for different sets of such sequences of matrices.

At the time $t = 0$ the matrix of sales standards $A(0) = A^{(k)}(0) \in A_1(t),\ k \in \overline{1,3}$ has the following concrete form:

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 1. The matrix $A(0)$

The initial values of the following parameters are specified:

$z = (200, 500, 200, 50, 50, 280, 40, 35)$;

$H = (10.75, 5.0, 13.0, 6.2, 5.0, 7.0, 8.2, 3.0)$;

$S = (150, 1827, 720, 38, 280, 68, 340, 50)$;

$u(0) = (0, 0, 0, 0, 0, 0, 0)$;

$x(0) = (140000000, 160000000, 35000000, 42000000, 300000000, 70000000, 140000000, 80000000)$;

$y(0) = (1636, 215, 250, 155, 42, 35, 1830)$;

$v = (60, 70, 40, 56, 52, 58, 40)$;

$r = (13.0, 10.4, 12.0, 15.2, 10.0, 6.0, 9.2, 9.6)$;

$c = (10.0, 10.0, 10.0, 10.0, 8.0, 0.1, 9.0, 0.1), q = 900000$;

And also with the given restrictions:

$T_{min} = 9600;\ T_{max} = 11040$.

The above described algorithm for finding the solution to the problem of optimal programmed terminal control is realized for a class of practical problems in the form of a computer modeling system in the Delphi 7 software environment.

RESULTS OF COMPUTER SIMULATION

The sales price matrix $A(t), t \in \overline{0, T-1}$ is the control action in the dynamical system (1)–(6) under consideration, and its admissible values were formed in accordance with the constraints.

Given the existing restrictions on the sales of various banking products by different categories of employees (in accordance with the specialization of employees), as well as data on the yield of relevant portfolios and labor costs for the sale of products, we will consider three variants of the values of the sales standards for each product per month presented in the respective matrices $A^{(k)}(t)$ $k \in \overline{1,3}$ with the maximum values of sales standards for each product (column). The form of each of these matrices is shown in Figures 2-4.

$$\begin{pmatrix} 19 & 3 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 2. The matrix $A^{(1)}(t)(CC)$

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 3. The matrix $A^{(2)}(t)(ML)$

$$\begin{pmatrix} 6 & 3 & 6 & 23 & 23 & 12 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 4. The matrix $A^{(3)}(t)(DC)$

The process of modeling the program control system (1)–(6) was considered on the time interval $\overline{0,T} = \overline{0,18}$ with the possibility of changing the permissible control action $\overline{u}(t) = \{u(t), A(t)\}$ at time instants $t \in \{0, 6, 12\} \subset \overline{0,18}$, assuming that the following conditions are satisfied: $\overline{u}(0) = \{u(0), A(0)\}$; $\forall t \in \overline{0,5}: \overline{u}(t) = \overline{u}(0) = \{u(0), A(0)\}$; $\forall t \in \overline{6,11}: \overline{u}(t) = \overline{u}(6) = \{u(6), A(6)\} \in \overline{\mathbf{U}}(6)$; $\forall t \in \overline{12,17}: \overline{u}(t) = \overline{u}(12) = \{u(12), A(12)\} \in \overline{\mathbf{U}}(12)$. Then for given initial data and constraints on the control action, a set of all admissible program controls is formed, defined by the set $\overline{\mathbf{U}}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\}_{k\in\overline{1,K_{18}}}$, consisting of $K_{18} = 9^{7\times2} = 9^{14}$ elements $\overline{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\}$, $k \in \overline{1,K_{18}}$, to which there corresponds the set of all admissible phase trajectories $\overline{x}^{(k)}(\cdot) = \{x^{(k)}(\cdot), y^{(k)}(\cdot)\} = \varphi_{\overline{0,18}}(\cdot; \overline{x}^{(k)}(0), \overline{u}^{(k)}(\cdot))$, $k \in \overline{1,K_{18}}$, the dynamical system under consideration, also consisting of $K_{18} = 9^{14}$ elements, where $\forall k \in \overline{1,K_{18}}: \overline{u}^{(k)}(\cdot) = \{\overline{u}^{(k)}(t)\}_{(t\in\overline{0,18})}$, $u^{(k)}(\cdot) = \{u^{(k)}(t)\}_{(t\in\overline{0,18})}$, $A^{(k)}(\cdot) = \{A^{(k)}(t)|\}_{(t\in\overline{0,18})}$; $\forall t \in \overline{0,18}: \overline{u}^{(k)}(t) = \{u^{(k)}(t), A^{(k)}(t)\} \in \overline{\mathbf{U}}(t)$.

The set of all admissible trajectories corresponds to the set of all admissible final phase states of a system of the form $\overline{x}^{(k)}(18) = \{x^{(k)}(18), y^{(k)}(18)\} = \varphi_{\overline{0.18}}(\cdot; \overline{x}^{(k)}(0), \overline{u}^{(k)}(\cdot))$. It is assumed that on the management gap $\overline{0,18}$ for each trajectory there are unified a priori specified initial sales standards, i.e. $\forall k \in \overline{1,3}: A^{(k)}(0) = A(0)$. For each k-th final value of the phase vector $\overline{x}^{(k)}(18) = \{x^{(k)}(18), y^{(k)}(18)\} = \varphi_{\overline{0,17}}(18; \overline{x}^{(k)}(0), \overline{u}^{(k)}(\cdot)$, $k \in \overline{1,K_{18}}$, in accordance with formula (5), we calculate value quality criteria of the process under consideration. Further, on the basis of the above algorithm for modeling the solution of the problem of optimal program management by the number of personnel and the sales of the Bank's Retail Unit, in the presence of an objective function of the form (5), the optimal program control is calculated: $u^{(e)}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \overline{\mathbf{U}}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\}_{(t\in\overline{0.17})} \in \overline{\mathbf{U}}(\cdot)$, the corresponding optimal trajectory $\overline{x}^{(e)}(\cdot) = \{x^{(e)}(\cdot), y^{(e)}(\cdot)\} = \varphi_{\overline{0,18}}(\cdot; \overline{x}^{(e)}(0), \overline{u}^{(e)}(\cdot))$, the real number $\mathbf{F}^{(e)}(T)$ is the optimal value of quality criterion for the surveys of the control process and the optimal value of the final phase vector $\overline{x}^{(e)}(18) = \{x^{(e)}(18), y^{(e)}(18)\} = \varphi_{\overline{0,18}}(18; \overline{x}^{(e)}(0), \overline{u}^{(e)}(\cdot)$, where $\overline{x}^{(e)}(0) = \overline{x}(0) = \{x(0), y(0)\}$.

The results of computer simulation are presented in Tables 1-2 and Figure 5.

TABLE 1: The optimal implementation of the management of the input of the number of employees

$u^{(e)}(t)$	t		
	0	6	12
$u_1^{(e)}(t)$	0	163	-179
$u_2^{(e)}(t)$	0	-21	-19
$u_3^{(e)}(t)$	0	25	27
$u_4^{(e)}(t)$	0	15	-17
$u_5^{(e)}(t)$	0	4	4
$u_6^{(e)}(t)$	0	3	3
$u_7^{(e)}(t)$	0	183	–201

TABLE 2: The optimal implementation of employee sales management standards

t	0	6	12
$A^{(e)}$ (t)	$A^{(2)}$ (ML)	$A^{(1)}$ (CC)	$A^{(1)}$ (CC)

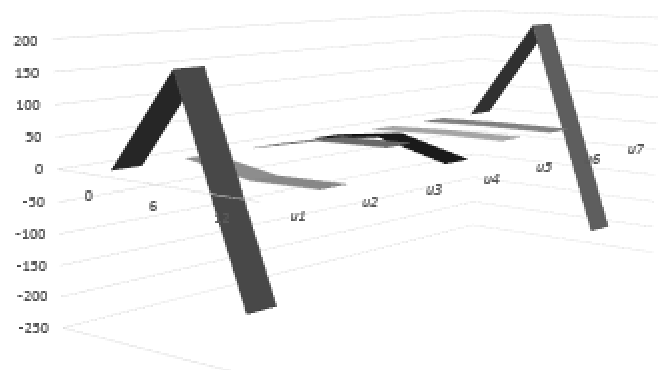

FIGURE 5. Realization of management of the input of the number of employees $u^{(e)}(t)$

Value of the Cost Income Ratio $\mathbf{\Phi}^{(e)} = \mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) = 52.30\%$

The graphs and tables below represent possible implementations of the control of the input of the number of employees $u^{(1)}(t)$, $u^{(2)}(t)$ at optimum realization of management by standards of sales of employees $A^{(e)}(t)$

TABLE 3: The implementation of the management of the input of the number of employees $u^{(1)}(t)$

$u^{(1)}(t)$	t		
	0	6	12
$u_1^{(1)}(t)$	0	-163	-147
$u_2^{(1)}(t)$	0	-21	-19
$u_3^{(1)}(t)$	0	-25	-22
$u_4^{(1)}(t)$	0	-15	-14
$u_5^{(1)}(t)$	0	-4	-3
$u_6^{(1)}(t)$	0	-3	-3
$u_7^{(1)}(t)$	0	-183	-164

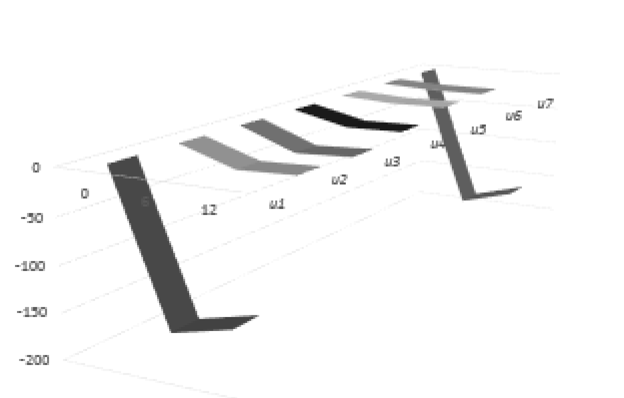

FIGURE 6. Realization of management of the input of the number of employees $u^{(1)}(t)$

Value of the Cost Income Ratio $\mathbf{\Phi}^{(1)} = \mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) = 52.99\%$

TABLE 4: The implementation of the management of the input of the number of employees $u^{(2)}(t)$

$u^{(2)}(t)$	t		
	0	6	12
$u_1^{(2)}(t)$	0	0	163
$u_2^{(2)}(t)$	0	0	21
$u_3^{(2)}(t)$	0	0	25
$u_4^{(2)}(t)$	0	0	15
$u_5^{(2)}(t)$	0	0	4
$u_6^{(2)}(t)$	0	0	3
$u_7^{(2)}(t)$	0	0	183

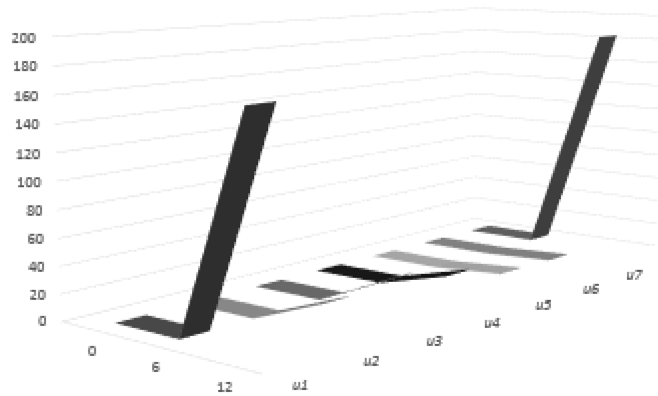

FIGURE 7. Realization of management of the input of the number of employees $u^{(2)}(t)$

Value of the Cost Income Ratio $\mathbf{\Phi}^{(2)} = \mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) = 52.78\%$

ANALYSIS OF THE RESULTS OF COMPUTER MODELING OF THE SOLUTION OF THE PROBLEM

As noted above, for the dynamical system (1)–(6) formed, the optimal path is optimal, corresponding to optimal program control, for which the value of the generalized quality criterion for the implementation of the process under consideration at the final instant is minimum in comparison with the analogous values for other admissible trajectories and program management. For a dynamic system with the parameters specified above, the optimal is:

- management in which the number of 2-nd category of employees decreases in the period of time $t = 6$ and and the number of other category is increase in similar period; in the period of time $t = 12$ the number of 3-rd, 5-th and 6-th categories of employees increase, other is decrease;
- management, which establishes sales standards with the maximum focus at the time $t = 0$ on mortgage loans, and in subsequent moments — on consume credits.

Taking into account the application of these control actions, the value of the quality criterion in this case is 52.30%, which is the absolute minimum.

CONCLUSIONS

This article proposes a new dynamic economic and mathematical model for optimizing the process of managing the number of employees and sales of the Retail Bank unit. The application of this model allows to solve one of the most important tasks of this process — the formation of the optimal number of employees who, with the necessary labor productivity, bring the bank the maximum profit. Similar mathematical models for economic systems are considered, for example, in [6]–[9].

The proposed economic-mathematical model in further studies will be complicated by expanding the phase vector of the system and including additional quality criteria for the implementation of the process under investigation.

Note that the dynamic economic and mathematical model formed in this article can serve as the basis for the development, creation and application of a comprehensive information and analytical system for supporting decision-making in banking. At the same time, the use of dynamic economic-mathematical models in banking significantly increases its efficiency, speeding up and optimizing the process of making managerial decisions, which makes the credit institution more competitive and flexible to changes in the market environment.

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REFERENCES

[1] R. A. Isaev. *Banking management and business-engineering*, INFRA-M, Moscow, 2013 [in Russian].

[2] N. E. Egorova, and A. M. Smulov. *Mathematical methods of financial analysis of banking activity*, Audit and financial analysis. 1998, 9. pp. 75–146. [in Russian].

[3] A. A. Fursa. *Methods for assessing the sufficiency of the number of personnel for servicing individuals' clients in commercial bank units* . Scientific and technical statements of SPbSPU Ser. Economic sciences, 2011, 2, pp. 160–163.[in Russian].

[4] A. F. Shorikov, and A. S. Filippova. *Application of dynamic economic and mathematical modeling to solve the problem of optimizing the process of managing the number of employees of a banking organization*, Vestnik UrFU. Series of Economics and Management, 2017, 5. pp. 779–802. [in Russian].

[5] A. S. Filippova. *Economic and Mathematical Modeling of the Dynamics of the State of Decision Support Systems in Banking Activity.* Vestnik Chelyabinsk State University, Chelyabinsk, Russia, 2015, pp. 103–111.[in Russian].

[6] V. A. Babenko. *Formation of economic-mathematical model for process dynamics of innovative technologies management at agroindustrial enterprises*,Actual Problems of Economics,, 139 (1). 2013. pp. 182-186. [in Russian].

[7] A. S. Filippova. *Multicriteria optimization of the structure of the number of employees of the commercial Bank's Retail Unit* . Vestnik of the Perm State Humanitarian and Pedagogical University, 3. Humanitarian and social sciences (electronic scientific journal). Issue. 1/2016. URL: http://vestnik3.pspu.ru/files/3-2016-1.pdf [in Russian].

[8] A. F. Shorikov, and E. Yu. Vinogradova. *Dynamic optimization of complex management of technological processes at the enterprise*, Izvestiya Ural State. econ. Universityl, 2007, 1(18). pp. 254–266.[in Russian].

[9] A. F. Shorikov, and E. S. Rassadina. *Mnogokriterialnaya optimization of the formation of the product range of the enterprise*, Regional economy. Scientific information-analytical journal, 2010, 2(22). pp. 189–196.[in Russian].